Thermal State Preparation via Rounding Promises

IBM **Quantum**

arXiv:2210.01670

Patrick Rall, Chunhao Wang, Pawel Wocjan

patrickjrall@ibm.com cwang@psu.edu pawel.wocjan@ibm.com

PennState

Main Result: A quantum algorithm for preparing thermal states by simulating thermalization

I. Overview

Given a Hamiltonian H, prepare its thermal state ρ_β.

Main Idea: If Nature can thermalize it, so can we!

Thermalization through Lindblad evolution

$$\sigma \longrightarrow e^{t_{\text{mix}}\mathcal{L}} \longrightarrow \rho_\beta$$

Input State — Thermal State

Lindblad simulation algorithm

Cleve, Wang arXiv:1612.09512, ICALP 2017

Li, Wang arXiv:2111.03240

Jump operators

$$\mathcal{L}(\sigma) = \sum_\omega \left[L_\omega \sigma L_\omega^\dagger - \frac{1}{2}\left(L_\omega^\dagger L_\omega \sigma + \sigma L_\omega^\dagger L_\omega \right) \right]$$

Input: Lindblad Oracle that specifies jump operators

$$|0\rangle \otimes |\psi\rangle \; -\mathcal{O}_\mathcal{L}\rightarrow \; \sum_\omega |\omega\rangle \otimes L_\omega |\psi\rangle$$

Davies Generator: Thermalizing Dynamics

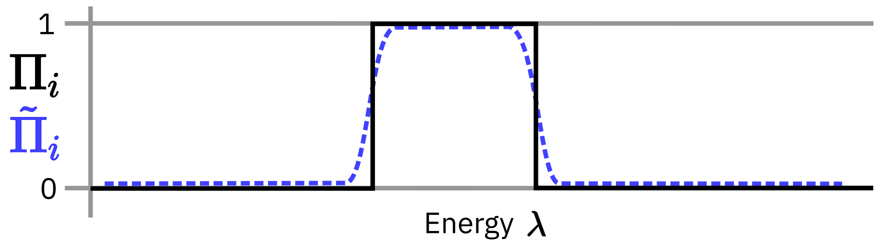
Filter function — Coupling operator — Energy projectors

$$L_\omega = \sqrt{G(\omega)} \sum_{\substack{i,j \\ \lambda_i - \lambda_j = \omega}} \Pi_i S \Pi_j$$

Objective: build Lindblad oracle for Davies generator.

II. Why Rounding Promises?

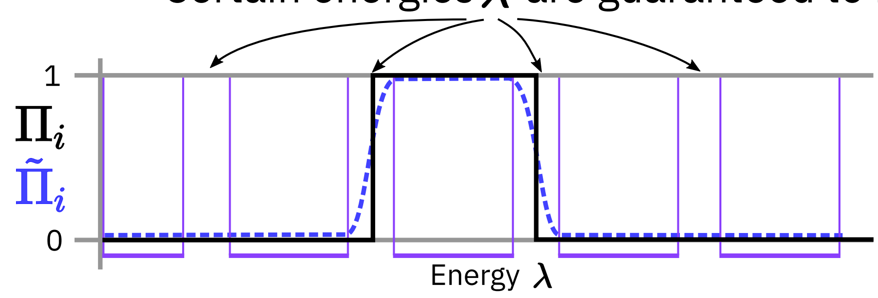
Phase Estimation's $\tilde{\Pi}_i$ are not accurate enough! (Approximate energy projector)



$|\Pi_i - \tilde{\Pi}_i| = \frac{1}{2}$, no matter how accurately you estimate

Solution approach: **Rounding Promises**

= Certain energies λ are guaranteed to not exist



$|\Pi_i - \tilde{\Pi}_i| \leq \delta$, accurate for energies "in the promise".

Rounding Promise → Accurate energy projectors $\tilde{\Pi}_i$

→ Accurate Lindblad oracle $\mathcal{O}_{\tilde{\mathcal{L}}}$

→ Accurate thermalization $e^{t\tilde{\mathcal{L}}}$

III. Is a "Promised" Thermal State Good?

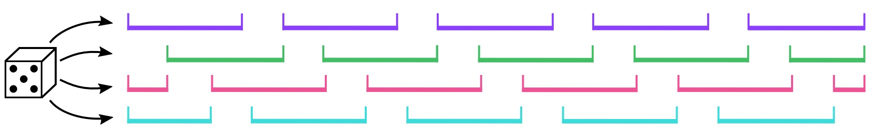
Original Hamiltonian H (no promise) →

"Promised" Hamiltonian $\hat{H}$ (bad energies removed)

$$\left|\rho_\beta^H - \rho_\beta^{\hat{H}}\right|_1 \approx 1$$

Missing energies: not a good approximation!

Solution: pick a **random promise**.



Each λ unlikely to be missing → $\left|\rho_\beta^H - \mathbb{E}_{\hat{H}}[\rho_\beta^{\hat{H}}]\right|_1 \leq \delta$

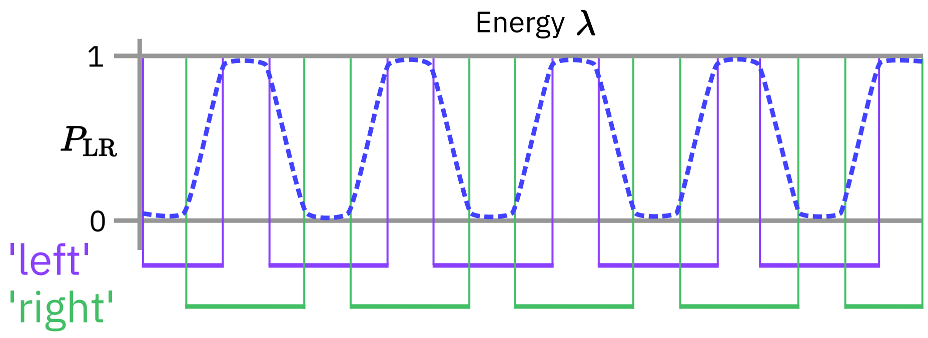
IV. Obtaining a Rounding Promise

$\hat{H}$ has no "bad" energies

↕

σ has no support on "bad" eigenvectors

Idea: perform a **measurement** that forces a promise



Measure POVM $P_{\text{LR}}, I - P_{\text{LR}}$ on input state σ → post measurement state satisfies left or right promise

V. Maintaining a Rounding Promise

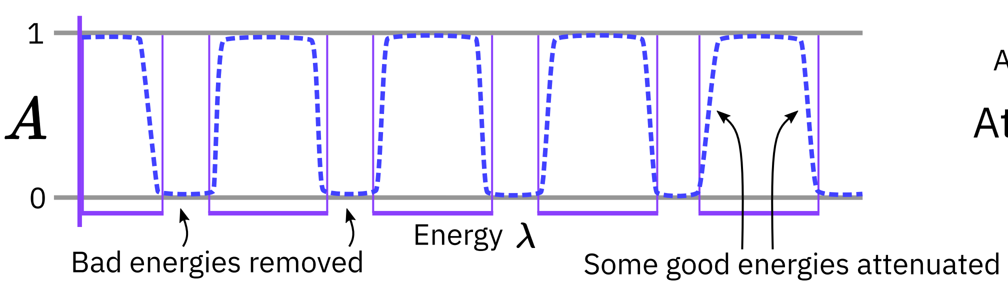
Purpose of coupling operator S → 'scramble' energy eigenstates

Problem: even if σ satisfies promise, scrambled state $S\sigma S^\dagger$ might not.

Solution: **'Attenuate'** S:

$$\hat{S} := ASA$$

$\hat{S}\sigma\hat{S}^\dagger$ satisfies promise.



VI. Performance

$$\tilde{O}\left(\tilde{t}_{\text{mix}} \cdot \gamma^{-1} \cdot \beta^3 / \varepsilon^7\right)$$

Attenuated mixing time — Attenuation strength

Attenuation slows scrambling rate.

Can attenuate less using a more expensive A.